Conceptual DFT: A paradigm for understanding chemical reactivity and selectivity

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MODERM CNRS Thematic School

Foreword

"**There remain real challenges in extracting insight** and not merely high quality numbers from electronic structure calculations..."

Martin Head-Gordon, *J. Phys. Chem.* **100**, 13213 (1996).

"It is nice to know that the computer understands the problem. But I would like to understand it too."

Eugene Wigner

"Accurate calculation is not synonymous with useful interpretation. To calculate a molecule is not to understand it."

Robert G. Parr

I. Introduction

Introduction: historical perspective

Timeline of bonding/reactivity models in chemistry:

Valence Bond Theory (L. Pauling), 1930's

- ↪ Resonance hybrid (L. Pauling and G. W. Wheland)
- ↪ Free valence index, electronegativity

Molecular Orbital Theory (R. Mulliken), 1950's

- ↪ Perturbation of MO (M. J. S. Dewar)
- ↪ Frontier MO (K. Fukui and R. Hoffmann)
- ↪ Charge/frontier control (G. Klopman and L. Salem)

Density Functional Theory (W. Kohn), 1970's

- ↪ **Conceptual DFT (R. G. Parr)**

Introduction: historical perspective

Common features of these different approaches:

- Trying to characterise, describe and rationalise the chemical reactivity
- development of models and concepts, for instance
  - electronegativity
  - chemical hardness (HSAB)
  - FMO theory

Introduction: electronegativity

According to IUPAC Gold Book:

Electronegativity:
" Concept introduced by L. Pauling[1] as the power of an atom to attract electrons to itself. There are several definitions of this quantity [...] a relative scale due to Pauling is used where dimensionless relative electronegativity differences are defined on the basis of bond dissociation energies, E_d, expressed in electronvolts:

$$\chi_r(A) - \chi_r(B) = (eV)^{-1/2}\sqrt{E_d(AB) - \frac{1}{2}\left[E_d(AA) + E_d(BB)\right]}$$

The scale is chosen so as to make $\chi_r(H) = 2.1$."

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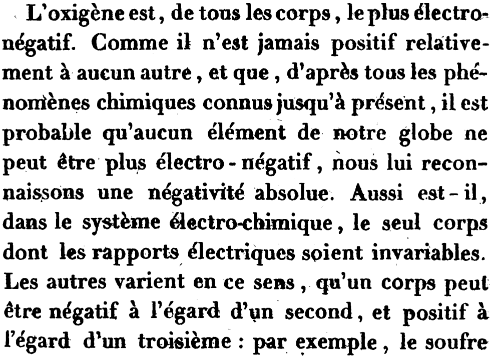
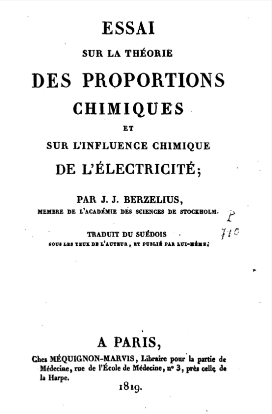
[1] Pauling, L. *J. Am. Chem. Soc* **1932**, *54*, 3570

Introduction: electronegativity

Well actually, Berzelius already proposed a similar concept in 1819



L'oxigène est, de tous les corps, le plus électro-négatif. Comme il n'est jamais positif relativement à aucun autre, et que, d'après tous les phénomènes chimiques connus jusqu'à présent, il est probable qu'aucun élément de notre globe ne peut être plus électro-négatif, nous lui reconnaissons une négativité absolue. Aussi est-il, dans le système électro-chimique, le seul corps dont les rapports électriques soient invariables. Les autres varient en ce sens, qu'un corps peut être négatif à l'égard d'un second, et positif à l'égard d'un troisième : par exemple, le soufre



ESSAI
SUR LA THÉORIE
DES PROPORTIONS
CHIMIQUES
ET
SUR L'INFLUENCE CHIMIQUE
DE L'ÉLECTRICITÉ;

PAR J. J. BERZELIUS,
MEMBRE DE L'ACADÉMIE DES SCIENCES DE STOCKHOLM.

P 710

TRADUIT DU SUÉDOIS
SOUS LES YEUX DE L'AUTEUR, ET PUBLIÉ PAR LUI-MÊME;

A PARIS,
Chez MÉQUIGNON-MARVIS, Libraire pour la partie de Médecine, rue de l'École de Médecine, nº 3, près celle de la Harpe.
1819.

Introduction: electronegativity

Many electronegativity scales:[2]

Author	**Support**	**Formula**	**Remark**
Pauling	Thermo.	$\chi(Y) - \chi(X) = 0.208\sqrt{\Delta}$	Relative
Daudel	Thermo.	$\chi(Y) - \chi(X) = 0.208\sqrt{\Delta} + m(\xi_X - \xi_Y)$	Relative + evolution
Mulliken	At. Spec.	$\chi(X) = \frac{1}{2}(I + A)$	Absolute
Jaffé	At. Spec.	$\chi(X) = \frac{1}{2}(I + A) + (I - A)q$	Absolute + evolution
Sanderson	Mean ρ(r)	$\chi(X) = Z_X/(4/3\pi R_X^3)$	Absolute

Note: the list is far from being complete... A unified view? CDFT!

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[2] Chermette, H.; Lissillour, R. *Actualité Chimique* **1985**, 4, 59.

Introduction: FMO theory and HSAB

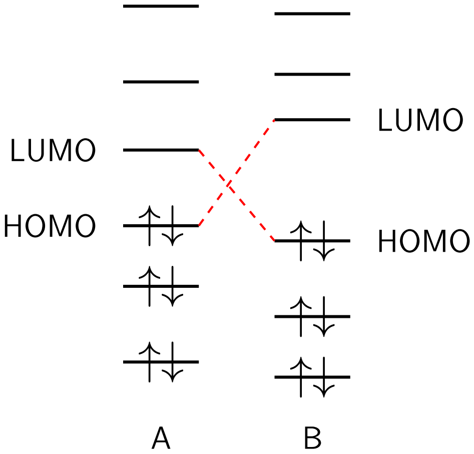
Interpreting and rationalising chemical reactivity and regioselectivity:

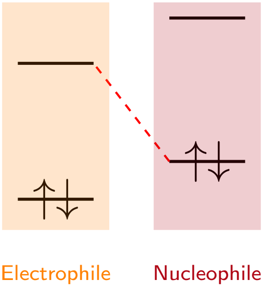
- Frontier Molecular Orbital Theory (K. Fukui)
- Hards and Soft Acids and Bases model (R. Pearson)

FMO theory: approximation from the MO theory

"Among all the interactions between orbitals, only those involving the interaction of the closest pair of occupied and unoccupied orbitals are considered, all the others being overlooked."

Introduction: FMO theory and HSAB





Introduction: FMO theory and HSAB

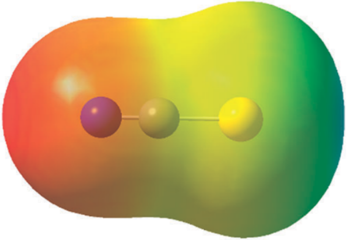
HSAB principles:

- Lewis acids (electrophiles) and bases (nucleophiles) are sorted out into two classes: Hard and Soft.
- **Hard** acids prefer to react with **hard** bases, *via* electrostatic interactions principally;
- **Soft** acids prefer to react with **soft** bases, principally through covalent (charge transfer) interactions.

Hard species: high charge density, not polarisable (ex: F^-)
Soft species: weaker charge density, highly polarisable (ex: H_2S)

Introduction: FMO theory and HSAB

Coordination chemistry of a "Janus Face" ligand, NCS^-

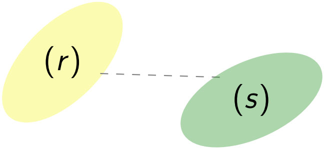


Red: high charge density (hard); Green: low charge density (soft). Calc.: B3LYP/6-31+G(d)

Experimentally: coordination by the N with hard metals, by the S atom with soft metals

Introduction: Klopman-Salem

Merging FMO theory and HSAB model: **Klopman-Salem Model**[3]



Interaction of two species r and s leads to an energy variation of

$$\Delta E = -q_r q_s \frac{\Gamma}{\epsilon} + \Delta solv + \sum_{m\ occ.} \sum_{n\ unocc.} \left[\frac{2(c_r^m)^2 (c_s^n)^2 \beta^2}{E_m - E_n} \right]$$
$$= E_{charge/charge} + E_{FMO/FMO}$$

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[3] (a) G. Klopman, *J. Am. Chem. Soc.*, **1968**, 90, 223–234.
(b) L. Salem, *J. Am. Chem. Soc.*, **1968**, 90, 543–552.

Introduction: Klopman-Salem

2 kinds of interactions between molecules

- charge controlled reactions: hard-hard interactions
  → index: Atomic Charges q
- Frontier controlled reactions (charge transfer): soft-soft
  → index: Coefficient of Frontier Orbitals c

Efficient model, but shares the issues of the FMO approach.
→ Rederiving within CDFT!
→ and expending: +polarisation

II. Foundation of CDFT

Foundation of CDFT

What is Conceptual DFT or DFT for Chemical Reactivity:

"Conceptual DFT concentrates on the extraction of chemically relevant concepts and principles from DFT."[4]

Starting point: **Hohenberg and Kohn theorems**

Hohenberg-Kohn I

The external potential v(r) is determined, within a trivial additive constant, by the electron density ρ(r).

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[4] P. Geerlings, F. De Proft and W. Langenaeker, *Chem. Rev.* **2003**, 103, 1793-1873.

Foundation of CDFT

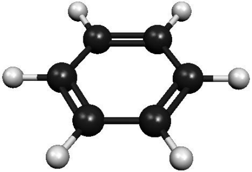
- from Schrödinger equation, E is a unique functional of Ψ.
- from HK-I, it is also a unique functional of $\rho(\mathrm{r})$
- implies that if Ψ contains all the information on the system, $\rho(\mathrm{r})$ **does too**.
- $\rho(\mathrm{r})$: **physical observable**, simple function of 3 space variables (Ψ: 3 N variables, with N the number of electrons).

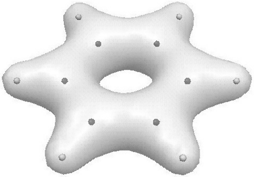
→ we may accurately describe our system basing on a simpler object

→ everything can be extracted from $\rho(\mathrm{r})$.

Foundation of CDFT

But can we get direct information from $\rho(\mathrm{r})$?





"Classical chemistry view":

- well located nuclei
- more or less defined bonds

"DFT view":

- blurred description
- located nuclei, but atoms?

→ Need to work a bit on the equations

Foundation of CDFT

On the way to our first C-DFT descriptor (Parr, 1978)...

Reminder: Energy in (Born-Oppenheimer) DFT

$$\mathrm{E_{molecule}} = \mathrm{E_{nuclei}} + \mathrm{E_{electrons}}$$

with

$$\begin{aligned}\mathrm{E_{electrons}} &= \mathrm{V_{ne}}\left[\rho(\mathrm{r})\right] + \mathrm{T}\left[\rho(\mathrm{r})\right] + \mathrm{V_{ee}}\left[\rho(\mathrm{r})\right] \\ &= \int \rho(\mathrm{r})\mathrm{v}(\mathrm{r})\mathrm{d}^3\mathrm{r} + \mathrm{F_{HK}}\left[\rho(\mathrm{r})\right].\end{aligned}$$

$\mathrm{F_{HK}}\left[\rho(\mathrm{r})\right] = \mathrm{T}\left[\rho(\mathrm{r})\right] + \mathrm{V_{ee}}\left[\rho(\mathrm{r})\right]$ is called the Universal Functional

Foundation of CDFT

Hohenberg-Kohn II

For a trial density $\tilde{\rho}(\mathrm{r})$, such that

$$\tilde{\rho}(\mathrm{r}) \geq 0 \quad \text{and} \quad \int \tilde{\rho}(\mathrm{r})\mathrm{dr} = \mathrm{N},$$

$$\mathrm{E}_0 \leq \mathrm{E}_\mathrm{v}\left[\tilde{\rho}(\mathrm{r})\right],$$

where $\mathrm{E}_\mathrm{v}\left[\tilde{\rho}(\mathrm{r})\right]$ is the energy functional. This is the exact equivalent of the variational principle in HF.

One can then try to find the real electron density by this variational principle.
→ minimisation of energy with conditions: Lagrange multiplier

Foundation of CDFT

Taylor expansion of the Energy:

$$dE_{\rho,v} = \int \left[\frac{\delta E}{\delta\rho(r)}\right] \delta\rho(r)dr + \int \left[\frac{\delta E}{\delta v(r)}\right] dv(r)dr + \ldots.$$

Minimising $E_{\rho,v}$ with respect to $\rho(r)$ means looking for the first derivative to vanish:

$$\int \left[\frac{\delta E}{\delta\rho(r)}\right] \delta\rho(r)dr = 0 \tag{1}$$

as $\int \rho\,(r)\,dr = N$, ergo $\int \delta\rho\,(r)\,dr = 0$ then for (1) to be respected,

$$\left[\frac{\delta E}{\delta\rho(r)}\right] = \mu$$

μ is independent of spatial coordinates

Foundation of CDFT

(alt) Minimising $\mathrm{E_v}\left[\rho(\mathrm{r})\right]$ for $\rho(\mathrm{r})$ integrating to N (at constant v(r)):

$$\delta\left\{\mathrm{E_v}\left[\rho(\mathrm{r})\right]-\mu\left[\int\rho(\mathrm{r})\mathrm{dr}-\mathrm{N}\right]\right\}=0 \tag{2}$$

where δ indicates a functional differential. We may expand:

$$\delta\mathrm{E_v}\left[\rho(\mathrm{r})\right]-\mu\,\delta\left[\int\rho(\mathrm{r})\mathrm{dr}-\mathrm{N}\right]=0 \tag{3}$$

$$\Leftrightarrow\ \delta\mathrm{E_v}\left[\rho(\mathrm{r})\right]-\mu\int\delta\rho(\mathrm{r})\mathrm{dr}=0 \tag{4}$$

$$\Leftrightarrow\ \delta\mathrm{E_v}\left[\rho(\mathrm{r})\right]-\mu\mathrm{dN}=0 \tag{5}$$

Foundation of CDFT

From equation (5) we thus can redefine μ:

$$\mu = \left(\frac{\partial \mathsf{E}}{\partial \mathsf{N}}\right)_{\mathsf{v(r)}} \tag{6}$$

μ is called the chemical potential, by analogy with thermodynamics

$$\mu = \left(\frac{\partial \mathsf{G}}{\partial \mathsf{n_i}}\right)_{\mathsf{P,T,n_j}} .$$

This is the variation of energy upon a variation in the number of electrons. Thus μ translates the stabilisation or destabilisation upon the addition or subtraction of one electron: it is related to **electronegativity**, and one can show that $\mu = -\chi$.

Foundation of CDFT

Showing for finite difference::

- Let's look at the subtraction of 1 electron. Derivative approximates to

$$\mu^{-} = \frac{\mathsf{E}(\mathsf{N}-1)-\mathsf{E}(\mathsf{N})}{\mathsf{N}-1-\mathsf{N}} = -\mathsf{I} \tag{7}$$

- For the addition of 1 electron:

$$\mu^{+} = \frac{\mathsf{E}(\mathsf{N}+1)-\mathsf{E}(\mathsf{N})}{\mathsf{N}+1-\mathsf{N}} = -\mathsf{A} \tag{8}$$

with I and A, resp., the ionisation potential and electron affinity. The FD value of μ is thus

$$\mu = \frac{1}{2}\left(\mu^{+}+\mu^{-}\right) = -\frac{1}{2}(\mathsf{I}+\mathsf{A}) = -\chi_{\mathsf{Mulliken}} \tag{9}$$

Foundation of CDFT

Note: we may even relate χ to the FMO theory. Assuming the MO diagram remains unperturbed under the addition or subtraction of 1 electron, one can write

$$\mathsf{I} = -\mathsf{E}_{\mathsf{HOMO}} \quad \text{and} \quad \mathsf{A} = -\mathsf{E}_{\mathsf{LUMO}}$$

by Koopman's theorem (or Janak if DFT). Thus

$$\mu_{\mathsf{FMO}} = \frac{1}{2}(\mathsf{E}_{\mathsf{HOMO}} + \mathsf{E}_{\mathsf{LUMO}}).$$

Foundation of CDFT

Note 2: Electron transfer in molecules
Let A and B be two molecular systems in interaction, *at constant geometry*, and μ_A and μ_B their chemical potential.

In the course of this interaction, some electron density can spontaneously be transferred from A to B or reciprocally. Calling $\Delta \mathsf{N}_\mathsf{A}$ and $\Delta \mathsf{N}_\mathsf{B}$ the variation in the number of electrons of A and B, we have $\Delta \mathsf{N}_\mathsf{A} + \Delta \mathsf{N}_\mathsf{B} = 0$ (fixed total number of electrons).

At first order, the energy stabilisation deriving from this interaction will be

$$\Delta \mathsf{E} = \mu_\mathsf{A} \Delta \mathsf{N}_\mathsf{A} + \mu_\mathsf{B} \Delta \mathsf{N}_\mathsf{B} = (\mu_\mathsf{A} - \mu_\mathsf{B})\, \Delta \mathsf{N}_\mathsf{A}$$

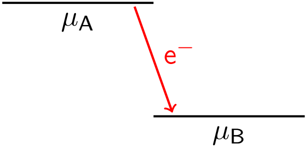
Foundation of CDFT

$$\Delta E = \mu_A \Delta N_A + \mu_B \Delta N_B = (\mu_A - \mu_B)\,\Delta N_A$$

Here, the electron transfer is spontaneous, so $\Delta E < 0$. If we chose $\mu_A > \mu_B$, then necessarily $\Delta N_A < 0$ and $\Delta N_B > 0$:

$$\Downarrow$$

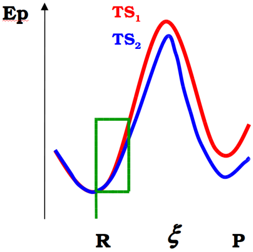
Electrons flow from high chemical potential towards low potential



III. A perturbation theory

A perturbation theory

Basic idea of C-DFT: study responses of the energy/density to small variations: perturbations



Restricted to early stages of reaction (extension by Hammond postulate), weak interactions...

A perturbation theory

Two variables stem from HK-I: N and v(r).

Descriptors: Energy responses[5]

$$\begin{tikzcd}
 & E \arrow[dl] \arrow[dr] & \\
\frac{\partial}{\partial N} & & \frac{\delta}{\delta v(r)}
\end{tikzcd}$$

→ Taylor development of the energy

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[5] Chermette, H. *J. Comput. Chem.* **1999**, *20*, 129.

A perturbation theory

$$\begin{aligned}
\mathrm{dE} = {} & \left(\frac{\partial E}{\partial N}\right)_{v(r)} \mathrm{dN} + \int \left[\frac{\delta E}{\delta v(r)}\right] \mathrm{d}v(r)\mathrm{d}r + \frac{1}{2}\left(\frac{\partial^2 E}{\partial N^2}\right)_{v(r)} \mathrm{dN}^2 \\
& + \mathrm{dN} \int \left[\frac{\delta^2 E}{\delta v(r)\partial N}\right] \mathrm{d}v(r)\mathrm{d}r + \frac{1}{2}\iint \left[\frac{\delta^2 E}{\delta v(r)\delta v(r')}\right] \mathrm{d}v(r)\mathrm{d}v(r')\mathrm{d}r\mathrm{d}r' \\
& + \frac{1}{6}\left(\frac{\partial^3 E}{\partial N^3}\right)_{v(r)} \mathrm{dN}^3 + \frac{1}{3}\mathrm{dN}^3 \int \left[\frac{\delta^3 E}{\delta v(r)\partial N^2}\right] \mathrm{d}v(r)\mathrm{d}r + \ldots
\end{aligned}$$

Successive derivatives may bear some chemical meaning

A perturbation theory

C-DFT chart of descriptors

$$\begin{tikzcd}
 & & & E \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & & & \\
 & & \mu \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & & \rho(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & & \\
 & \eta \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & & f(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & & \chi(r, r') \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(\mathbf{r})"] & \\
\gamma & & \Delta f(r) & & \xi(r, r') & & \chi_2(r, r', r'')
\end{tikzcd}$$

μ chemical potential, η hardness, γ hyperhardness
$f(r)$ Fukui functions, $\Delta f(r)$ Dual Descriptor, $\chi(r, r')$ linear response

Grand canonical ensemble

- Up to now: systems with constant N
- problem: chemistry usually implies electron exchanges
- utility in deriving descriptors for variable N (avoid dealing with complexes)
- in thermodynamics, moving from constant V to constant P: Legendre transform (exemple: $H = U + PV$)
- same approach can be unfolded here: transform E with respect to N

Grand canonical ensemble

- we can define a grand potential Ω:

$$\Omega = E - \mu N$$

- and thus the grand canonical ensemble, with natural variables μ and $v(\mathrm{r})$;
- allows to define a similar chart of descriptors, fitted to compare systems with different N

Grand canonical ensemble

$$\begin{tikzcd}
 & & & \Omega \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & & & \\
 & & -N \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & & \rho(r) \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & & \\
 & S \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & & s(r) \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & & -s(r,r') \arrow[dl, "\partial/\partial\mu"'] \arrow[dr, "\delta/\delta v(r)"] & \\
-S^{(2)} & & \Delta s(r) & & \xi_{GC}(r,r') & & \chi_{2,GC}(r,r',r'')
\end{tikzcd}$$

S: softness, $s(r)$: local softness, $s(r,r')$: softness kernel, $\Delta s(r)$ grand-canonical DD.

III. Global descriptors

Global descriptors

$$\begin{tikzcd}
 & & & E \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & & \\
 & & \mu \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \rho(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \\
 & \eta \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & f(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \chi(r, r') \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & \\
\gamma & & \Delta f(r) & & \xi(r, r') & & \chi_2(r, r', r'')
\end{tikzcd}$$

Yield information on the chemical/physical properties of the system **considered as a whole**

Global descriptors

Chemical potential: see previously

Chemical hardness:[6]

$$\eta = \left(\frac{\partial^2 E}{\partial N^2}\right)_{v(r)} = \left(\frac{\partial \mu}{\partial N}\right)_{v(r)} \tag{10}$$

Exercise: Show that, in the ground state at constant geometry, $\eta \geq 0$.

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[6] Parr, R.G. and Pearson, R. G. *J. Am. Chem. Soc.* **1983**, *105*, 7512–7516.

Global descriptors

Chemical potential: see previously

Chemical hardness:[6]

$$\eta = \left(\frac{\partial^2 \mathrm{E}}{\partial \mathrm{N}^2}\right)_{\mathrm{v(r)}} = \left(\frac{\partial \mu}{\partial \mathrm{N}}\right)_{\mathrm{v(r)}} \tag{10}$$

Exercise: Show that, in the ground state at constant geometry, $\eta \geq 0$.

Solution: According to HK-II, energy is minimal in the ground state. At constant geometry (v(r) kept fixed), this implies a zero first-order derivative and a positive curvature (convexity) of the curve $\mathrm{E} = \mathrm{f(N)}$, thus a positive second order derivative.

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[6] Parr, R.G. and Pearson, R. G. *J. Am. Chem. Soc.* **1983**, *105*, 7512–7516.

Global descriptors

Chemical meaning of η? Finite difference:

$$\eta \approx \frac{\Delta}{\Delta N}\left(\frac{\Delta E}{\Delta N}\right) = \frac{E(N+1)-E(N)}{1} - \frac{E(N)-E(N-1)}{1}$$
$$= -A + I$$

which expresses the resistance to a charge transfer.[7] Indeed, for

$$S + S \longrightarrow S^{+} + S^{-}$$

$\Delta E = I - A$.

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[7] Parr, R.G. and Yang, W. *Density Functional Theory of Atoms and Molecules*, Oxford Univ. Press 1989.

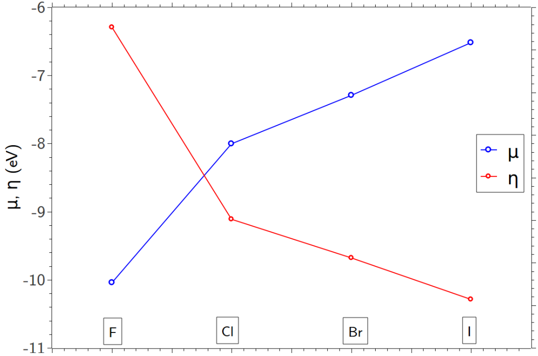
Global descriptors

- small or zero η means electrons can easily flow from S to S
  $\rightarrow$ soft species
- large η values mean electrons do not tend to move from S to S
  $\rightarrow$ hard species

η is thus directly linked to the HSAB concepts.

Global descriptors

Application on the halogens



Global descriptors

Closely related descriptor:

$$\mathsf{S} = \left(\frac{\partial \mathsf{N}}{\partial \mu}\right)_{\mathsf{v(r)}} = \frac{1}{\eta}$$

is called chemical softness.

Global descriptors

We can use hardness to characterise the optimum number of transferred electrons between two molecules A and B in interaction.

Expressing the energy variation up to the second order we have

$$\begin{aligned}\Delta E = \left(\frac{\partial E}{\partial N_A}\right)\Delta N_A + \frac{1}{2}\left(\frac{\partial^2 E}{\partial N_A^2}\right)\Delta N_A^2 \\ + \left(\frac{\partial E}{\partial N_B}\right)\Delta N_B + \frac{1}{2}\left(\frac{\partial^2 E}{\partial N_A^2}\right)\Delta N_B^2\end{aligned}$$

Global descriptors

Recalling that the electron count is fixed,

$$\Delta N = \Delta N_A + \Delta N_B = 0.$$

which leads to

$$\Delta E = (\mu_{\mathrm{A}} - \mu_{\mathrm{B}})\Delta \mathrm{N}_{\mathrm{A}} + \frac{1}{2}(\eta_{\mathrm{A}} + \eta_{\mathrm{B}})\Delta {\mathrm{N}_{\mathrm{A}}}^2.$$

The optimum value of ΔN_A can then be found from the condition

$$\left(\frac{\partial \Delta E}{\partial \Delta N_A}\right)_{\mathrm{v(r)}} = 0,$$

Global descriptors

$$\Leftrightarrow \left(\frac{\partial \Delta \mathrm{E}}{\partial \Delta \mathrm{N_A}}\right)_{\mathrm{v(r)}} = 0 = \mu_\mathrm{A} - \mu_\mathrm{B} + (\eta_\mathrm{A} + \eta_\mathrm{B})\Delta \mathrm{N_A}$$

$$\Leftrightarrow \Delta N_A = \frac{\mu_\mathrm{B} - \mu_\mathrm{A}}{\eta_\mathrm{B} + \eta_\mathrm{A}}$$

Equivalent of the Ohm's law $\mathrm{I} = \Delta \mathrm{V}/\mathrm{R}$

- hard species: large denominator, weak charge transfer
- soft species: small denominator, huge charge transfer

Global descriptors

Another global descriptor: Electrophilicity index

Consider a system (μ, η) surrounded by an electron reservoir of zero chemical potential and infinite softness. The second order energy variation under an electron transfer with this reservoir is

$$\Delta \mathrm{E} = \mu \Delta \mathrm{N} + \frac{1}{2}\eta \Delta \mathrm{N}^2.$$

The system is saturated when $\Delta\Delta \mathrm{E}/\Delta \mathrm{N} = 0$, then

$$\frac{\Delta\Delta \mathrm{E}}{\Delta \mathrm{N}} = \mu + \eta \Delta \mathrm{N} = 0.$$

Global descriptors

The max number of electrons the system can acquire is:

$$\Delta \mathrm{N}_{\mathrm{max}} = -\frac{\mu}{\eta} \tag{11}$$

thus the energy variation is

$$\Delta \mathrm{E} = -\frac{\mu^2}{2\eta} = -\omega < 0.$$

This electron gain ($\Delta \mathrm{N} > 0$) is always stabilising, the energy gain can thus be used to characterise the propensity of the system to gain electrons. ω is called the **electrophilicity index**.[8]

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[8] R.G. Parr *et al.*, *J. Am. Chem. Soc.* **1999**, *121* (9), 1922–1924

IV. Local and non-local descriptors

Local and non-local descriptors

$$\begin{tikzcd}
 & & & E \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & & \\
 & & \mu \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \rho(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \\
 & \eta \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & f(r) \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & & \chi(r, r') \arrow[dl, "\partial/\partial N"'] \arrow[dr, "\delta/\delta v(r)"] & \\
\gamma & & \Delta f(r) & & \xi(r, r') & & \chi_2(r, r', r'')
\end{tikzcd}$$

These descriptors will yield an insight on the local reactivity (and also on non-local effects).

Local and non-local descriptors

Fukui function

$$\mathrm{f(r)} = \left(\frac{\partial \rho(\mathrm{r})}{\partial \mathrm{N}} \right)_{\mathrm{v(r)}} = \left[\frac{\delta \mu}{\delta \mathrm{v(r)}} \right]_{\mathrm{N}} \tag{12}$$

- translates the variations of electron density to a modification in the number of electrons
- or the sensitivity of the chemical potential to an external perturbation

Local and non-local descriptors

Actually, 3 different Fukui functions can be defined

$$f^{+}(r) = \left(\frac{\partial \rho(r)}{\partial N}\right)^{+}_{v(r)} \tag{13}$$

$$f^{-}(r) = \left(\frac{\partial \rho(r)}{\partial N}\right)^{-}_{v(r)} \tag{14}$$

$$f^{0}(r) = \frac{1}{2}(f^{+}(r) + f^{-}(r)) \tag{15}$$

Left/right derivatives are **inequivalent** (and $f^0(r)$ has little to no meaning).

Local and non-local descriptors

Assuming the MO diagram remains unperturbed by subtraction or addition of one electron, one can show within the FD approximation that

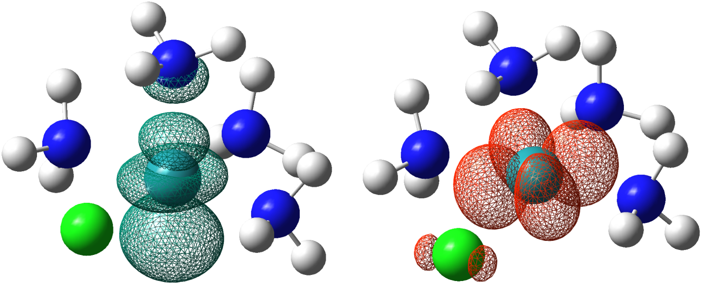
$$\mathrm{f}^{+}(\mathrm{r}) \approx \rho_{\mathrm{LUMO}}(\mathrm{r}) \tag{16}$$

$$\mathrm{f}^{-}(\mathrm{r}) \approx \rho_{\mathrm{HOMO}}(\mathrm{r}) \tag{17}$$

Exercise: prove this.

Local and non-local descriptors

Fukui functions yield information on regioselectivity.
Ex: $[RuCl(NH_3)_4]^+$, f^+ (left) and f^- (right)[9]



Marked electrophilicity on the Ru vacant site

---

[9] Level: B3LYP/6-311+G(d) + SDD(Ru).

Local and non-local descriptors

Dual Descriptor:[10][11]

$$\Delta f(r) = \left(\frac{\partial^2 \rho(r)}{\partial N^2}\right)_{v(r)} = \left[\frac{\delta \eta}{\delta v(r)}\right]_N \tag{18}$$

- translates the reorganisation of electron density upon charge transfer
- or the local response of hardness to an external perturbation

---

[10] Morell, C.; Grand, A.; Toro-Labbé A. *J. Phys. Chem. A* **2005**, 109,205

[11] Morell, C.; Ayers, P.W. ; Grand, A.; Chermette, H. *Phys. Chem. Chem. Phys.* **2011**,13, 9601.

Local and non-local descriptors

Approximations of the DD:

$$
\begin{aligned}
\Delta f(r) = \left(\frac{\partial^2 \rho(r)}{\partial N^2}\right)_{v(r)} &\approx \rho_{N+1}(r) + \rho_{N-1}(r) - 2\rho_N(r) && (19)\\
&\approx f^+(r) - f^-(r) && (20)\\
&\approx \rho_{LUMO}(r) - \rho_{HOMO}(r) && (21)
\end{aligned}
$$

(19) and (20) stem from the FD scheme, and (21) comes from the frozen orbital hypothesis.

Exercise: prove this.

Local and non-local descriptors

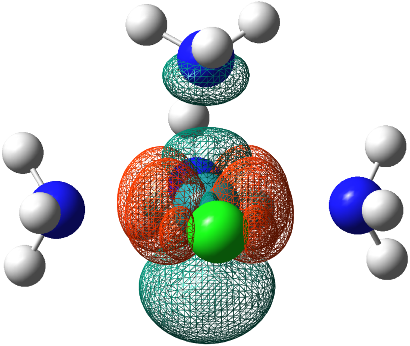
From these different formulas:

- link with regioselectivity is clear
- DD integrates to 0 over whole space (**Exercise**: proof.)
- DD < 0 for any region prone to cede electron density (nucleophile)
- DD > 0 for electrophilic sites

Despite being formally a third order descriptor, it is meaningful and actually better than Fukui functions for describing regioselectivity (well-defined mathematically).

Local and non-local descriptors

On the previous example:



We retrieve the same information as contained within the 2 Fukui functions

Re-conciliating Fukui Function and Dual Descriptor

Change in density due to charge transfer reads at second order:

$$\delta\rho\left(r\right)=\left(\frac{\partial\rho\left(r\right)}{\partial N}\right)_{v}dN+\frac{1}{2}\left(\frac{\partial^{2}\rho\left(r\right)}{\partial N^{2}}\right)_{v}dN^{2} \qquad (22)$$

Therefore:

$$\delta\rho\left(r\right)=\frac{1}{2}\left(f^{+}\left(r\right)+f^{-}\left(r\right)\right)dN+\frac{1}{2}\left(f^{+}\left(r\right)-f^{-}\left(r\right)\right)dN^{2} \qquad (23)$$

for $dN=+1$:

$$\delta\rho\left(r\right)=f^{+}\left(r\right)$$

for $dN=-1$:

$$\delta\rho\left(r\right)=f^{-}\left(r\right)$$

Local and non-local descriptors

A non local descriptor: **linear response**

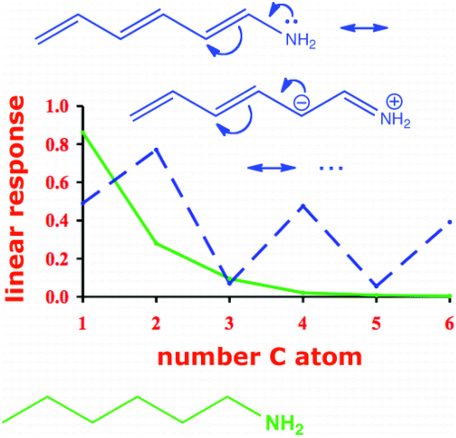
$$\chi(\mathsf{r},\mathsf{r}') = \left[\frac{\delta\rho(\mathsf{r})}{\delta \mathsf{v}(\mathsf{r}')}\right]_{\mathsf{N}} \tag{24}$$

Direct meaning of this descriptor is less obivous: response of electron density on one point r under a perturbation in another point r′.

→ will somehow translate **correlation** and delocalisation effects

Local and non-local descriptors

Example:[12]



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[12] Sablon, N.; De Proft, F.; Geerling, P.; *J. Phys. Chem. Lett.* **2010**, *1* (8), 1228–1234.

Local and non-local descriptors

LRF can furthermore be used to compute other quantities:

- polarisability tensor α_{ij}, (i,j=x,y,z)[13]

$$\alpha_{ij} = \iint i\chi(r, r')j \, drdr'$$

- electron density polarisation induced by perturbation δv:[14]

$$\delta\rho(r) = \int \chi(r, r')\delta v(r')dr'$$

- and associated energy stabilisation (polarisation energy):[15]

$$\delta E^{(2)} = \int \delta\rho(r)\delta v(r)dr$$

---

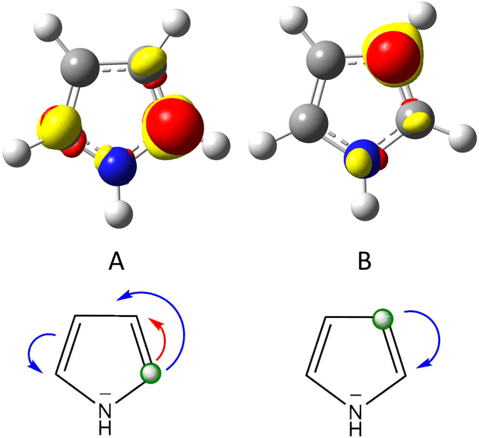
[13] P. Geerlings *et al*; *Chem. Soc. Rev.* **2014**, *43*, 4989–5008.

[14] F. Guégan *et. al.*, *J. Phys. Chem. A* **2020**, *124* (4), 633–641.

[15] Index (2): second-order perturbation response.

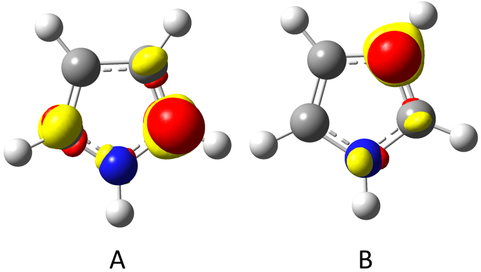
Local and non-local descriptors

- electron density polarisation: allows to map how ρ reorganises under a perturbation (link with Lewis structures and mesomery)



Local and non-local descriptors

- polarisation energy: quantification



$$\delta E^{(2)}(A) = -0.93 \text{ eV/e}, \ \delta E^{(2)}(A) = -0.70 \text{ eV/e}$$

$\rightarrow \alpha$ carbon more polarisable, likely more reactive: OK with exp.

Local and non-local descriptors

Note: these local and non-local descriptors can be integrated on volumes of $\mathbb{R}^3$:

- for instance volumes delimiting an isosurface ("reactivity domains" of the DD)[16]
- or atomic volumes (choose the partition you like :D)

affording numerical values (quantitative/semi-quantitative approach)

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[16] Tognetti, V.; Morell, C.; Joubert, L. *J. Comput. Chem.* **2015**, *36*, 649-659.

V. New chemical principles

New chemical principles

Conceptual DFT non only allows to (re)define descriptors and concepts for chemistry, it sometimes allows to propose (or validate) new principles.

- Sanderson electronegativity equalisation principle;[17]
- Maximum Hardness principle.[18]

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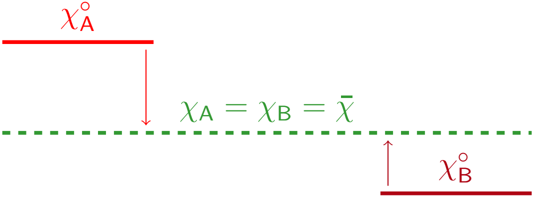
[17] Sanderson, R. T. *Science* **1951**, *114*, 670.

[18] Pearson, R.G. *J.Chem.Educ.* **1987**, 561

New chemical principles

Sanderson Principle of Electronegativity Equalisation

"when atoms of initially different electronegativity combine to form a compound, all atoms become adjusted to the same intermediate electronegativity within the compound"



χ_A°

$\chi_A = \chi_B = \bar{\chi}$

χ_B°

New chemical principles

Proof: Consider two systems A and B in interaction at constant geometry. At first order we have

$$\mathrm{d}E = \left(\frac{\partial E}{\partial N_A}\right)_{v(r),N_B} \mathrm{d}N_A + \left(\frac{\partial E}{\partial N_B}\right)_{v(r),N_A} \mathrm{d}N_B = 0.$$

If the system is isolated, $\mathrm{d}N_A + \mathrm{d}N_B = 0$, hence

$$\begin{aligned} & \left(\frac{\partial E}{\partial N_A}\right)_{v(r),N_B} \mathrm{d}N_A = -\left(\frac{\partial E}{\partial N_B}\right)_{v(r),N_A} \mathrm{d}N_B \\ \Leftrightarrow\ & \left(\frac{\partial E}{\partial N_A}\right)_{v(r),N_B} \mathrm{d}N_A = \left(\frac{\partial E}{\partial N_B}\right)_{v(r),N_A} \mathrm{d}N_A \end{aligned}$$

New chemical principles

$$\mu_{\mathsf{A}} = \left(\frac{\partial \mathsf{E}}{\partial \mathsf{N}_{\mathsf{A}}}\right)_{\mathsf{v}(\mathsf{r}),\mathsf{N}_{\mathsf{B}}} = \left(\frac{\partial \mathsf{E}}{\partial \mathsf{N}_{\mathsf{B}}}\right)_{\mathsf{v}(\mathsf{r}),\mathsf{N}_{\mathsf{A}}} = \mu_{\mathsf{B}}.$$

□

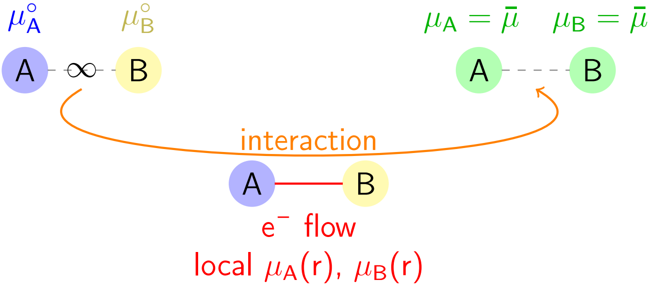
But how does this equalisation process occur?
From the definition of μ (Lagrange multiplier) we can also write

$$\mu = \frac{\delta \mathsf{E}}{\delta \rho(\mathsf{r})} = \mathsf{v}(\mathsf{r}) + \frac{\delta \mathsf{F}_{\mathsf{HK}}\left[\rho(\mathsf{r})\right]}{\delta \rho(\mathsf{r})} \tag{25}$$

μ **is a global descriptor based on two local functions!**

New chemical principles

Concept of "local chemical potential":[19]



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[19] C. Morell, P. W. Ayers, A. Grand, H. Chermette, *Phys. Chem. Chem. Phys.* **2011**, *13*, 9601–9608.

New chemical principles

Principle of Maximal Hardness

"It seems to be a rule of nature that molecules rearrange themselves to be as hard as possible"

Demonstration by Parr and Chattaraj[20], quite technical (mixes out-of equilibrium statistical physics and 0 K quantum chemistry), very strong hypotheses (constant μ and v(r)).

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[20] Parr, R.G.; Chattaraj, P.K., *J.Am.Chem.Soc.* **1991**, 1854

New chemical principles

Intuitive basis:

- a good nucleophile should have a high energy HOMO
- a good electrophile should have a low energy LUMO

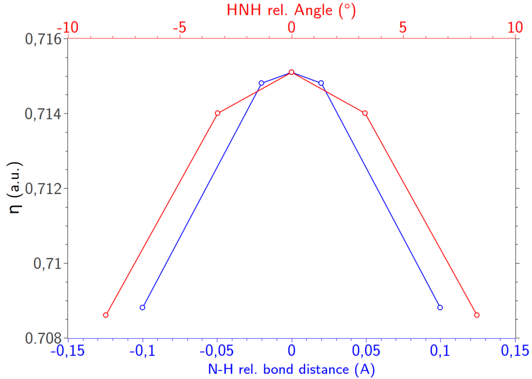
A non reactive species (end product) should thus be neither nucleophilic nor electrophilic: low energy HOMO, high energy LUMO, and as such

$$\eta \approx \mathrm{E}_{\mathrm{LUMO}} - \mathrm{E}_{\mathrm{HOMO}}$$

should be the highest possible to ensure non-reactivity.

New chemical principles

Example of the ammonia molecule:[21]



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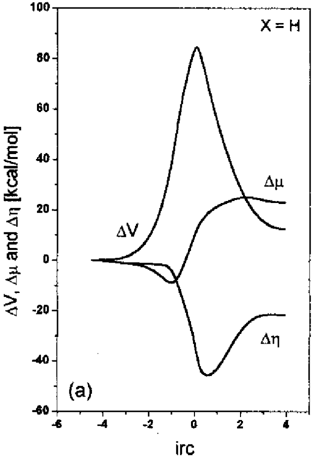
[21] Pearson, R.G.; Palke, W.E. *J. Phys. Chem.* **1992**, *96*, 3283-3285.

New chemical principles

PMH and chemical reactivity:[22]



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[22] Perez, P.; Toro-Labbé, A. *J. Phys. Chem. A*, **2000**, 1557.

VI. Conclusion

Conclusion

Conceptual DFT:

- Rigorous mathematical grounds (distribution theory)
- "Minimalist" theory: two founding theorems only (+ quantum mechanics postulates)
- Sound physical basis
- Interpretative theory
- Predictive theory
- Unifying (extending) different frameworks (FMO, HSAB, Sanderson)
- not restricted to DFT: one just needs an electron density!

Perspectives

Beyond this lecture:

- additional variables: temperature, electric/magnetic fields, mechanical pressure...
- spin-related quantities
- excited-states extension of the theory...